

Social Choice in Medieval Europe

Iain McLean

Nuffield College, Oxford

Haidee Lorrey

Berklee College of Music

Josep Colomer

University Pompeu Fabra

**For Colloque « Histoire des Mathématiques Sociales », Paris,
November 2007**

Abstract

We take institutions seriously as both a rational response to dilemmas in which agents found themselves and a frame to which later rational agents adapted their behaviour in turn. Medieval corporate bodies knew that they needed choice procedures. Although the social choice advances of ancient Greece and Rome were not rediscovered until the high middle ages, the rational design of choice institutions predated their rediscovery and took some new paths. Both Ramon Llull (ca 1232-1316) and Nicolaus of Cusa (a.k.a Cusanus; 1401-64) made contributions which had been believed to be centuries more recent. Llull promotes the method of pairwise comparison, and proposes the Copeland rule to select a winner. Cusanus proposes the Borda rule, which should properly be renamed the Cusanus rule.

Voting might be needed in any institution ruled by more than one person, where decisions could not simply be handed down from above. Medieval theologians no doubt believed that God's word was handed down from above; but they well knew that they often had to decide among rival human interpretations of it. The Church faced its own decision problem every time a new Pope needed to be elected. Bodies not directly in the hierarchy of the Church had to evolve their own decision procedures. The chief such bodies were commercial and urban corporations; religious orders; and

universities.

The disagreement between Llull and Cusanus raises the issue: should voting be regarded as a method of aggregating judgments or as a method of aggregating interests? In the former interpretation (only), voting procedures are a solution to a problem of approximate reasoning.

There is an unknown, true state of affairs (for medieval thinkers, divine will). A voting procedure aggregates unreliable individual perceptions of the will of God to a more reliable group judgment of it. In the rougher world of Cusanus, and probably of electors to the papacy and to Dogeships, only at most lip service is paid to the will of God, and voting is a process of aggregating interests.

Social Choice in Medieval Europe

1. Introduction

We take institutions seriously as both a rational response to dilemmas in which agents found themselves and a frame to which later rational agents adapted their behaviour in turn. Modalities of thought differ; rationality does not. We aim to show that the institutions we discuss represent rational responses to problems. Medieval corporate agents knew that they needed choice procedures. Although the social choice advances of ancient Greece and Rome were not rediscovered until the high middle ages, the rational design of choice institutions predated their rediscovery and took some new paths.

Although we normally think of voting as a mechanism for expressing tastes or preferences, there is an alternative framework in which it represents *judgments*. This was the framework not only for medieval social choice but also for much of classical social choice up to and including Condorcet. Ramon Llull saw voting precisely as a mechanism for approximate reasoning. Humans had an imperfect understanding of God's will. They differed in their understanding of the correct course of action, where the correct course of action was to carry out God's will. Voting was a procedure for aggregating these imperfect individual judgments into a more reliable group judgment. His follower Nicolaus Cusanus proposed a different choice procedure, for rival electors from different parts of Europe, with different territorial interests, to choose an Emperor. In Llull's interpretation (only), voting procedures are a solution to a problem of approximate reasoning. There is an unknown, true state of affairs (for Llull, the will of God). In the rougher world of Cusanus, and probably of electors to the papacy and to Dogeships, only at most lip service is paid to the will of God, and voting is a process of aggregating interests.

The strategy of this paper is as follows. Section 2 introduces and interprets the texts of Llull and Cusanus. Sections 3 and 4 review medieval practice in elections of popes and officials of monastic orders. Section 5 concludes, reviewing the history of the two rival conceptions of voting in medieval Europe.

2. Theory: Llull and Cusanus

The classical Greek contribution to the theory and practice of social choice lies in the development of juries and other random choice procedures of judgment aggregation. In democratic Athens in the era of Pericles, the governing institutions were Assembly, Council, and juries. The Assembly was the meeting of all citizens. The executive was the Council, whose membership was chosen by lot and rotation, so that any citizen might be president of Athens for a day. Juries were bodies of (typically) 501, 1001, or 1501 members, numbers being odd to avoid ties. Voting played a relatively minor role, the most common recorded case being votes on proposals to ostracise (banish) citizens.

Voting was a more developed institution in republican Rome. Pliny the Younger discusses parliamentary procedure in the Roman Senate (Farquharson 1969 *passim*; Riker 1986 chapter 7; McLean and Urken 1995, chapter 2). He is clearly writing in a context of commonly understood rules. However, he is thinking more in an *interests* than in a *judgments* framework, since the context is what we now label strategic, or sophisticated, voting. Pliny tried to justify his novel stratagem (of replacing binary voting by ternary, in the vain hope of gaining a strategic advantage) in terms of appeals to ancient parliamentary authority.

The theorists we consider here made an entirely fresh start. After Pliny, social choice theory did not appear again in the Western world until the sudden emergence of

sophisticated themes in the work of Ramon Llull (ca. 1232--1316). Llull, and probably Nicolaus of Cusa (Cusanus; Niklaus von Kues, 1401-64), did not know the Greek and Latin writings on social choice. But Llull worked at the frontier of Christian and Islamic scholarship. Thus the issue may have been discussed during the golden age of Arabic scholarship between the seventh and thirteenth centuries C.E.; but we have not found any evidence for that. However, progress beyond Pliny requires algebra and some knowledge of combinations and permutations, and these were Arabic inventions. Medieval Europe saw little discussion of democracy. The framework within which people either thought or had to pretend that they thought was one in which a choice either conformed to the will of God or it did not: a binary choice. Llull and Cusanus saw that the case of multiple candidates threw up new problems.

Llull was a native of Palma (Mallorca), which in his time was part of a Catalan economic empire with links across the western Mediterranean, including mainland Spain, recently under Islamic rule, and north Africa, still under Islamic rule. Llull wrote copiously in Catalan, Arabic, and Latin. At the age of about thirty he became a devout Christian and devoted the rest of his life to missionary work and theology. He wrote poetry and a novel (the first in any Western European language), and copious writings in mathematics and logic. In his autobiography, written in 1311, he wrote that when he was converted he had a vision that God had called him to write "a book, the best book in the world, against the errors of unbelievers" (Llull 1311, in Bonner 1985, I:15). This was to be the *Ars Generalis* with which Llull struggled for the rest of his life, in between journeys to North Africa to convert the Moors and equally unsuccessful visits to successive popes urging them to set up language schools for missionaries. He frequently introduced mathematical arguments for the truth of Christianity into his theological works. His theory of voting appears three

known places: his novel *Blanquerna*, written in Catalan between 1282 and 1287; a newly discovered paper *Artifitium electionis personarum* (discovered by Perez Martinez 1959, but first discussed by Hägele and Pukelsheim 2001; date unknown but before 1283) and a short paper entitled *De arte electionis*, written in 1299. We describe first the more picturesque text, then the more scholarly ones.

Blanquerna is the beloved son of Evast and Aloma. When he reaches the age of eighteen, he decides to become a hermit despite his mother's anguished pleas to stay with his parents. His mother sends Natana to him in the hope of persuading him to stay; instead, he persuades Natana to renounce her possessions as well and enter a nunnery. In due course she becomes its abbess; meanwhile, Blanquerna becomes successively a monk, an abbot, a bishop (reluctantly), and the pope, before renouncing everything again and becoming a hermit. The story gives Llull the opportunity to introduce homely dialogs illustrating the deadly sins and the virtues of the Christian life. In one of his anecdotes, the Abbess has died, and the nuns are deciding what to do.

All the sisters wanted to elect their abbess by their usual electoral method, but

Natana said that she had heard of a new electoral method, which consisted in art

[in other words, Llull's General Art] and figures. (McLean and Urken 1995, p. 71).

Natana told the twenty sisters that they should first elect seven electors, each to nominate (presumably seven) names excluding herself. The seven electors should compare the candidates with each other

according to four conditions, namely, which of them best loves and knows God, which of them best loves and knows the virtues, which of them knows and hates most strongly the vices, and which is the most suitable person.

She goes on to describe the case of nine candidates (viz., the seven electors and two

outsiders). The electors should compare the candidates two by two, and for each pair determine which they judge to be the more God-loving, virtuous, vice-hating, and suitable:

Therefore, taking this number as an example, 36 cells [*cambres*] will be produced in which the votes of each candidate will appear. The candidate to be elected should be the one with the most votes in the most cells. (adapted from McLean and Urken 1995, pp. 71-3).

The theory of elections recurs in later chapters of *Blanquerna*, as when Blanquerna is elected abbot "according to the manner of election whereby Natana had been elected abbess." Later still, he is proposed for a bishopric. He does not want it because it would mean giving up the contemplative life. Most of the electors nevertheless vote for him on the advice of the retiring bishop, but his enemy the archdeacon "opposed the holding of an election according to the art". One takes place "without the art," but it leads to a dispute, the majority electing Blanquerna and the minority the archdeacon. Both sides go to Rome, where the pope rules in favor of the reluctant Blanquerna. Thus people who oppose the correct art of elections come to a suitably sticky end (Peers 1926, chaps. 24, 60, and 67). Hägele and Pukelsheim (2001) date *Artifitium electionis personarum* to between 1274 and 1283. It is in a Vatican MS which they translate into English for the first time. Like the later *De Arte Eleccionis*, it opens with a matrix for pairwise comparisons of candidates, in this case with 16 candidates and hence 120 pairs. The crux of the procedural description is in these sentences, describing how each pair is to be scored:

Et omnes responderint et eligerint prout eis uidebitur[. F]iat vnus punctus in littera attribuata illi person[a]e qu[a]e plures uoces habuerit. Qui punctus fiat ipsi littere in qualibet figurarum existentium in locis diuersis. Si uero vna habuerit tot uoces ut altera fiat in qualibet littera ipsius camere punctus vnus et hoc in

qualibet figurarum.

And all shall respond, and shall elect as it appears [fit] to them. [Then] a point is placed by the letter assigned to the person who has the most votes. Such a point is marked in each of the figures existing at the distinct locations. If now one [person] has as many votes as another, then a point is placed by both letters of this cell, and this [is done] in each of the figures [in the vote matrix]. (Transcription and translation by Hägele and Pukelsheim 2001 with minor edits by IM and HL)

The electoral procedure in *De Arte Eleccionis* was devised, Llull tells us, at Paris on 1 July, 1299. In his autobiography he complained that nobody understood him when he lectured in Paris because of his "Arabic way of speaking" (Bonner 1985, vol. 1, pp. 29, 38.) *De Arte Eleccionis* contains a proposed method of election, similar to that in *Artifitium electionis personarum*, except that it does not use all the pairwise comparisons in the vote matrix.

The electoral method in *Blanquerna* is a two-stage procedure. Like Condorcet and the US Federalists five centuries later, Llull seems to wish to compromise between democracy and giving a more decisive voice to better qualified electors. The election is to be made on multiple (four) criteria. Llull may have realized that multiple-criterion decision making can lead to difficulties in aggregating from individual to social orderings. And it is a method of exhaustive pairwise comparisons. Votes are to be placed in 36 *cambres* (cells).

These represent the 36 combinations of two candidates from nine---as it would now be written $n(n - 1)/2$ for $n = 9$. Llull has Natana state that "the candidate to be elected should be the one with the most votes in the most cells". How is this phrase to be interpreted?

There are two natural interpretations, one of which makes *Blanquerna* an anticipation of the Borda rule and the other an anticipation of the Copeland rule. (Riker 1982 p. 79; McLean and Urken 1995, p. 18; Klamler 2005).

On the first interpretation, the phrase "in the most cells" is redundant, since each candidate will have votes in just eight cells. These votes are simply summed, and the candidate with the highest aggregate is elected. This is, as is now well known, exactly a Borda count in which zero points are awarded for a last place, one for a second-to-last, and so on up to $n-1$ for a top place. Borda pointed out this equivalence in his paper of 1770 (in McLean and Urken 1995, p. 87). On this interpretation, the following passage about ties refers to ties in the Borda count:

One of the sisters asked her, "If it turns out that some candidates have as many votes as each other in the cells, what procedure does the art recommend?" Natana replied, "The art recommends that these two or three or more should be judged according to art alone. It should be found out which of these best meets the four aforementioned conditions, for she will be the one who is worthy to be elected".

The Copeland rule has regard to the number of majorities each candidate has, not to their size, individually or in aggregate. It selects the candidate who wins the largest number of contests. If there is no cycle, the Copeland winner is the same as the Condorcet winner. If there is a top cycle, there is no Condorcet winner and a set of Copeland winners numbering three or more. Is this what Llull meant? On this interpretation, the whole phrase, "The candidate to be elected should be the one with the most votes in the most cells" is then an exact instruction to select the Copeland winner, reading the first "most" as "more"; and the passage about ties is an instruction on how to select a unique winner from the Copeland set if that contains more than one member. But note that the Copeland set cannot contain just two members unless there are ties on individual pairs arising from abstention, individual indifference, or an even number of voters, none of which Llull seems to allow in this text since he seems to insist on an odd number of voters, each with a strong ordering.

There is no such ambiguity in either *Artifitium electionis personarum* (AEP) or in *De Arte Eleccionis*. (DAE) The passage translated above from AEP seems to clinch matters in favor of Copeland. The winner of each pair is counted and a mark put against the winner's name. The passage translated makes it clear that this is a group, not an individual, mark. That individual is then given one point (*unus punctus*) for each majority win. The winner must be the candidate with the largest number of *puncti*, that is, the Copeland winner. If there is a tie, both candidates are awarded one point, so that there could be a Copelandoid set of size 2. The procedure Llull recommends in the two papers, unlike that described in *Blanquerna*, is a Condorcet pairwise comparison procedure. It uses matrix notation, previously thought to have been first used by C. L. Dodgson (Lewis Carroll) nearly six centuries later. Because the winning candidate must have beaten at least one other, it cannot select a Condorcet loser, and if a Condorcet winner exists, it will select him or her. The AEP procedure could, but the DAE procedure could not, detect the existence of cycles because in DAE not every comparison in the matrix is actually used in selecting the winner. The fact that Llull makes this (to us) retrograde step in the later procedure suggests that he did not discover cycling.

Thus Llull does not deserve the scornful treatment he gets in modern histories of mathematics and logic. Combinatorics, which he was probably one of the first mathematicians in the West to import from the Arab world, fascinated him endlessly and fueled the magnificent but impossible dream of the General Art. Llull believed that applying successive pairwise combinations of virtues could lead one to "demonstrating the truth of the holy Catholic faith through the use of necessary reasons to those who are ignorant of it" (Bonner 1985, 69). This led Donald Michie to label Llull "one of the most inspired madmen who ever lived" (Gardner 1982, ix). Martin Gardner has written that

Llull's life was "much more fascinating than his eccentric logic. . . . Llull's mistake . . . was to suppose that his combinatorial method had useful applications to subject matters where today we see clearly that it does not apply" (Gardner 1982, xiv, 18). However, the application to voting rules is an entirely appropriate application of the mathematics of combinations, not repeated until 1785. *Providing* that voting is regarded as a procedure for aggregating imperfect individual judgments into a more reliable group judgment, exhaustive pairwise comparison is an appropriate method.

Nicolaus Cusanus read *De arte electionis* and may have been the transcriber of the only known copy: Cusanus was born in 1401 beside the river Moselle. He studied first at Heidelberg, then at Padua, where he gained his doctorate in 1423, then at Cologne. Padua was one of the leading intellectual centers of Europe, and Llull's mathematical and theological works were on the curriculum there (Sigmund 1963, 22-35). Llull had a dangerous reputation: anybody whose ideas were as hard to follow as his risked being suspected of heresy. But the intellectual climate in northern Italy was more open than elsewhere. Cusanus was active in the conciliar movement of his time. The Council of Constance (1414-1417) addressed the Great Schism in the papacy that had lasted since 1378; it succeeded in ousting all three of the current contenders for the title of pope and electing one of its choice. It featured a weighted voting scheme (voting by nations) to ensure that the Italian electors did not carry the day by sheer force of numbers. Most council members were bishops, and Italy had the largest number of bishoprics.

Cusanus's *De concordantia catholica* was written while he was attending the Council of Basel, which opened in 1431; Cusanus was an active member from 1432 to 1434. *De concordantia catholica* defends the rights of councils to elect popes, and it discusses voting procedures for electing a Holy Roman Emperor in chapters 36 to 38 of Book III.

Cusanus first discusses the need to prevent *practicās absurdissimas et inhonestissimas* (the most absurd and dishonest practices) and notes that because particular electors come from particular districts, *turpiter foedatae electiones per iniustas pactiones fieri dicuntur* (elections are said to be disgracefully rigged by means of unjust pacts) (McLean and Urken 1995, pp. 77-8).

Cusanus' scheme is just the Borda count, giving 1 for a last place and so on up to n for a top place. Although Cusanus knew of the existence of *Blanquerna*, he had probably not read it. A mention of it exists in a handlist of Llull's work in Cusanus's library (Honecker 1937b, 570--1), but the work itself does not, and the library is believed to have survived complete. The only known copy of *De Arte Eleccionis* comes from this library, transcribed in what is believed to be Cusanus's handwriting. Thus Cusanus knew Llull's Condorcet scheme of public voting; but he proposed instead a Borda scheme with secret voting.

We believe that Cusanus' rejection of Llull is deliberate. We refer to the phrase *quoniam omnes comparationes omnium personarum et omnes mixturae et syllogismi per unumquemque ex electoribus factibiles in hoc modo includuntur*. "For this method takes account of all comparisons of candidate to candidate---in whatever groupings or combinations---that any elector can make," in Cusanus's paragraph 540. The use of *syllogismi* in this sense is highly unusual. A syllogism involves at least three elements (major premise, minor premise and conclusion). In using the word, Cusanus may have had in mind that a voter's transitive ordering of three elements (I prefer A to B and B to C ; therefore I prefer A to C) is fully captured by the Borda count but not always by the scheme of *De Arte Eleccionis*. The theory of voting involves pairwise comparisons, as

Llull had seen. But it does not only involve pairwise comparisons.

Both writers wish to eliminate strategic voting, but they make opposite recommendations. Llull, in *Blanquerna*, writes about the members of a religious order voting selecting their own leader. The electors are all known to each other and must continue to live together after the vote. A voter will then be constrained by her fellow voters' knowledge of her preferences. In general, this is the argument for open voting in committees where the members must trust one another if business is to be done. It is an argument traditionally accepted in the direct democracy, with open voting, of some Swiss cantons (Barber 1985). Cusanus writes about a body of electors meeting once only and suspicious of one another's strategic voting intentions before the election starts. Increasing the amount of information about others' votes available to each voter increases the opportunities and incentives for strategic voting of a logrolling kind. It was presumably in part to prevent this that the Council of Constance had voted by nations. Llull sees voting as an aggregation of judgments; Cusanus, writing from bitter experience, sees it as an aggregation of interests. Cusanus goes on to show that his method may also be applied to votes on propositions when more than two possibilities exist and contrasts it with the simple binary procedure in use in Venice for yes or no propositions and for elections (Sigmund 1991, 307, 580). The Venetian procedure for electing a doge, in use between 1268 and 1797, has been regarded as pointlessly complicated. Lines (1986), however, shows that "a great deal of the tedious complication . . . served only to ensure the impossibility of forecasting just who would be in the Quarantuno [the 41 electors]" (Lines 1986, 156). The actual election stage was approval voting with a lower bound. The electors voted on each of the ten candidates separately and could vote in favor of or against as many as they chose; the winner would

be the candidate with the most favorable votes, as long as he got more than 25.

Thus we find four of the main solution concepts of modern voting theory---the Borda rule, the Condorcet principle, the Copeland rule, and approval voting---in use in medieval Europe. We should not be surprised to find intelligent discussion of voting schemes first appearing in the West in the Middle Ages. The experience of the Great Schism in the papacy showed that elections could not be restricted to two candidates. All orders of monks, nuns and friars had to make their own rules for electing their superiors; since they were entirely separate from the ordinary parish clergy, there was no hierarchy except their own to choose their leaders. This is the situation addressed by Llull.

3. Practice: the papacy

Unanimity was first conceived as the only rule that could reveal God's will, but it led to frequent deadlocks, conflicts and schisms. It was replaced with the qualified majority rule of two-thirds in 1179. However the particular mixture of devices that were adopted by the Church during the late Middle Ages did not tend to produce quick and consensual decisions, but frequently led to uninformed coalition-building with surprising results.

In early centuries, the Pope was elected by a multicameral college. First, the lay members of the Roman Church proposed candidates. Second, the clergy proceeded to reduce or to enlarge that list. Finally, the sixteen bishops of the Roman province met and decided. This sequence was reflected in the motto of pope Leo I (440-61) *vota civium, testimonia populorum, honoratum arbitrium, electio clericorum*. This frequently produced conflicts and schisms. Before the Emperor had officially accepted Christianity, there was at least one simultaneous election of two different popes (in 250, after 18 months of deadlock). Afterwards, elections of pairs of popes by different factions of the Church provoked intervention of Roman troops in 366 and 418. These conflicts put the Church under

political protection. The emperor Honorius ruled in 420 that if two popes were elected, neither would be valid and a new election would be called in which divine judgment, as revealed by unanimity (*divinum iudicium et universitatis consensus*), would be required. The primacy of the Pope over political powers was doctrinally asserted in pope Gregory VII's bull *Dictatus Papae* (1075). But enforcing it required intensive legislative activity, starting with Gratian (1139-40), followed by four enlarged canon law codes in 100 years. The Church required an orderly succession of its monarch. New rules emerged from successive decisions on partial aspects of the question. The first was a papal bull of 1059 which excluded laymen from the election of the Pope (Nicolaus II, *In nomine Domini*). The role of the Emperor was once again reduced to mere acceptance of the Church's decision. Although secular rulers tried to continue exercising their veto-right against certain candidates for pope, this now had to be implemented by way of some faction of cardinals. Royal or imperial 'anti-popes' ceased to exist after 1122. This did not avert conflict. Three 12th-century elections produced a total of eight antipopes to only nine 'official' popes in less than fifty years. Even without direct imperial appointments, these schisms emerged because the voters could not reach unanimous agreement. Several procedures to manufacture unanimity were implemented, known as 'acclamation', 'scrutiny', and 'compromissum'. Elections by 'acclamation' were rare and, even according to some participants, enthusiastic and threatening roaring of crowds induced them, not initial coincidence of voters around a single candidate. (For the election of Gregory VII in 1073, so reported by the pope himself, see Robinson, 1990: 59-60). 'Compromissum' consisted in delegating the decision to a small commission when unanimous agreement could not be reached. However, delegation should be adopted by unanimity of those entitled to vote, specifying the rule to be followed by the delegates, and

this was not a frequent resource either. The most common procedure was 'scrutiny', that is, voting, but new intellectual devices had to be implemented to create apparent unanimity where it did not exist. The most discussed of these was the *sanior et maior pars*, the 'sounder and greater part'. *Sanior* referred to the priority given to cardinal-bishops, to candidates' merit and to voters' merits, zeal or dignity (including age or seniority in post, and hierarchy). All these qualities were considered factors for the choice of 'the best' candidate. But the 'sanior pars' often did not coincide with the 'maior pars'. In elections of bishops or abbots such disputes were usually submitted to some arbitrator, such as the metropolitan bishop or even the Pope, but no such arbiter existed for papal elections (See authorities cited in Colomer and McLean 1998, p. 2).

Such conflicts led to the adoption of two-thirds majority rule by Pope Alexander III in 1179. The rule of two-thirds had previously been used in the election of some abbots. Two-thirds and other qualified-majority rules were also used in several Italian communes of the Middle Ages. Alexander III spent six months in Venice in 1177, forging reconciliation with the Emperor Frederick Barbarossa who had supported the 'anti-pope'. Alexander decreed.

Concerning the election of the supreme pontiff.

...We decree, therefore, that if, by chance, some hostile man sowing discord among the cardinals, full concord cannot be attained with regard to constituting a pope; and, with the two thirds which agree, the other third be unwilling to agree, or presume of itself to ordain someone else: he shall be considered Roman pontiff who shall be elected and received by two thirds.....

Moreover if anyone is elected to the office of pope by fewer than two thirds-- unless greater concord is attained, he shall by no means be accepted, and shall be

subject to the aforesaid penalty if he is unwilling to humbly abstain. From this, however, let no prejudice to the canonical and other ecclesiastical decrees arise, with regard to which the opinion of the greater and the sounder part [*maior et sanior pars*] should prevail; for when a doubt arises with regard to them, it can be defined by the judgement of a higher power. But in the Roman church, special decrees are made because recourse cannot be had to a higher power. (Doeberl 1889-94, iv. p 253)

It seems clear that the basic aim of the qualified majority of two-thirds was to induce the formation of a sufficiently large coalition of cardinals. A two-thirds winner would tend to require a previous negotiation between supporters of different candidates, probably around compromise solutions. As some contemporary analyses noted, once a candidate was elected, the losers would need to persuade a majority of the winner's original supporters to change their mind. Faced with this requirement, it was reasonable to expect that the losing coalition would not fight on (see also Saari, 1994: 15-16).

Caplin and Nalebuff (1988) have shown that the rule of 64% guarantees a single winner under conditions of 'concavity' in voter preferences. This means that, when more voters prefer intermediate candidates than the average of those favoring extremes, there exists an unbeatable proposal, and furthermore no cycles are possible. In general, the majority rule needed to avoid cycles and ensure existence of an unbeatable proposal in a n -dimensional issue space is no higher than $1 - [n/(n+1)]^n$. This ratio is 55% for two dimensional spaces, 57% for three-dimensional spaces, and, being increasing in n , its limit is $1 - (1/e)$, which is just under 64%. In a simple aggregation of interests, there can be up to many dimensions as candidates. This means that the 2/3 rule used to elect the Pope produces a stable outcome if Caplin and Nalebuff's conditions hold.

After adopting the rule of two-thirds, popes and canonists tended to agree that the 'maior pars' is always, by definition, also the 'sanior pars'. As pope Pius II summarized (on his own election in 1458): "What is done by two thirds of the sacred college, that is surely of the Holy Ghost, which may not be resisted" (in Gragg and Gabel, 1959: 88). The 2/3 rule is, for the first time, explicitly defended as a judgmental aggregation rule of approximate reasoning - even if it was reached by a completely different route.

The qualified majority requirement produced the desired stability effects but it had predictable consequences. The electors in 1216, 1241, 1243, 1261, 1265 and 1268-70 took several months to reach a decision, having to resort to commissions in several cases. In two of these elections, (1216 and 1243), the civil authorities reacted to cardinals' slowness by locking them up. In 1241 the head of civil administration in Rome locked them up in an old unhygienic building, guarded by police, but he only elicited a decision by threatening to have the corpse of the dead pope exhumed and shown publicly in full papal regalia after two years of the vacancy. In 1270, when two years had passed without an agreement, the public besieged the cardinals in the episcopal palace, removed the roof of the palace and allowed nothing but bread and water to be sent in. A new pope was elected on this occasion by *compromissum* after a record vacancy of 34 months (Vauchez, 1990: 522-3). Thus the two-thirds rule produced efficacious and rather stable outcomes, at the price of long delays in decision-making. This is now recognized as a classic trade-off in social choice.

The experiences of locking cardinals up led pope Gregory X to adopt a new procedure for their seclusion, known as the Conclave (Latin: 'with-key'), which was approved by the council of Lyon in 1274 (*Ubi periculum*). It aimed to obtain a quick decision, and to prevent strategic maneuvering in the election of the Pope. Similar institutions had been

established in the Dominican constitution of 1228, as well as in communes such as Venice and Piacenza, respectively in 1229 and 1233 (Ruffini Avondo, 1925; Ullmann, 1972).

The cardinals gathered together, each with no more than one servant, in a closed papal palace, whose doors were walled up and watched by soldiers; they were to lead a life in common in a single room; to have no communication with the outer world; food was to be supplied to them through a guarded window; the menu was restricted from the fourth day on and reduced to bread, water and wine after the ninth day; the cardinals received no income until they reached a collective decision. Although some of these provisions were later softened, they created strong and increasing incentives for the cardinals to reach a common decision. Many cardinals fell ill and several died in conclave, precipitating agreement among the remaining participants. All side-payments, coercion or explicit pacts between cardinals were forbidden under penalty of excommunication and annulment of the election; they must keep silence during the election and afterwards. These rules made exchanges and formation of large coalitions very difficult and often promoted agreed outcomes on the basis of the immediate, apparent appeal of some candidate rather than on careful evaluation of his merits or religious fervor.

The first papal election under this procedure, in 1276, was made in one single day. The following popes suspended the application of this procedure, whereupon long delays reappeared: more than seven months in 1277, six months in 1281, almost eleven months in 1288, and 27 months in 1292-94. This evidence of conclaves' efficacy moved the pope elected in 1294, Celestine V, to re-establish it. Successful conclaves of one or a few days have become normal since then, including in 2006. For the rules of conclave see the authorities cited by Colomer and McLean 1998, p. 14.

4. Practice: monastic orders

Monastic orders faced the same problem as the papacy. They were not directly subject to papal control at all times, and were required to choose their own leaders and secure an unbroken succession whenever a leader died. The papal bull *Exiit qui seminat*, promulgated by Pope Nicholas III in 1279, contains provisions for the Franciscan order that illustrates just how reliant on self-determination and tradition the succession issue could be. As the bull notes, outlining the accepted procedure for choosing a new master of the order,

Besides the friars of the aforesaid order doubting in regard to that which is said in the rule, that with the decease of the minister general there is to be an election of a successor by the ministers provincial and guardians¹ (*custodes*) in the Pentecost chapter, whether it is fitting that the multitude of all the *custodes* come together to the general chapter, or whether, so that everything be managed with greater tranquillity, it may be able to suffice that some from each province, who would vote in the name of others, would take part, We give this answer that namely the *custodes* of each province are to appoint one from [among] themselves, whom they are to send with their minister provincial on their own behalf to the chapter, committing their votes and powers to the same, because, when they have appointed [him] by themselves, even We reckon a statute of this kind to have been approved, because also [Our] predecessor, Gregory IX, in a case of this kind is said to have responded in this manner. (*Exiit qui seminat*, Nicholas III. Our translation from the Latin text transcribed from the Registers of Nicholas III, p. 232-241, #564)

Many of the later monastic rules exhibit more sophisticated choice procedures.

The rule of St Benedict (6th century) was the first to be codified, and all subsequent orders until the Dominicans (early 13th century) followed the Benedictine rule with minor tweaks. All new orders had to deal with the succession crisis following the death of their founder. The rule of St. Benedict makes it clear that if officials were simply appointed, rather than elected by a majority, there might ensue dissent within an abbey--or the entire order.

It happens all too often that the constituting of a Prior gives rise to grave scandals in monasteries. For there are some who become inflated with the evil spirit of pride and consider themselves second Abbots. By usurping power they foster scandals and cause dissensions in the community. Especially does this happen in those places where the Prior is constituted by the same Bishop or the same Abbots who constitute the Abbot himself. What an absurd procedure this is can easily be seen; for it gives the Prior an occasion for becoming proud from the very time of his constitution, by putting the thought into his mind that he is freed from the authority of his Abbot. (Rule of St. Benedict. St. Benedict's rule for monasteries, tr. Leonard Doyle OSB, Collegeville, MN, 2001. Chapter 65: On the Prior of the Monastery, Apr. 22 - Aug. 22 - Dec. 22)

Another example is that of the Gilbertines, founded by Gilbert of Sempringham some time before 1147, when he travelled to the Cistercian headquarters at Citeaux in order to derive a constitution from theirs. As the Gilbertines were a double order, of both monks and nuns, he had to devise a more complex choice procedure. It was assumed in typical Benedictine procedure that decisions would be unanimous, but in the case of a difference, a majority of 3 to 1 sufficed, any huge differences were referred to the magister (head of the order). (The

Gilbertine constitution is in University of Oxford, Bodleian Library, MS Douce 136).

The most elaborate constitution was that of the Dominicans, first written in 1216, and revised in 1228 (Galbraith 1925, especially 5, 33, 46, 64, 103, 114, 226-36). They give a much greater role than earlier constitutions to internal democracy. In the early years of the Order, friars acquired the suffrage immediately upon profession. Later, friars had to wait first one, then two, then ultimately 4 years after profession to receive voting privileges. Later in the 13th century, this concept of democracy gained the authority of Aquinas, through his rediscovery and commendation of Aristotle.

Question 105. The Various Forms of Government, and their Fusion in the Government Given by God to the Jews. (Art. 1)

With respect to the right ordering of power in a city or nation, two points must be considered: the first is that all should in some respect participate in the government.... So the best ordering of power within a city or a kingdom is obtained when there is one virtuous head who commands over all; and who has under him others who govern virtuously; and when, furthermore, all participate in such government, both because all are eligible, and because all participate in the election of those who rule.... For Moses and his successors governed their people as sole heads over all: but they elected to their assistance seventy two Elders according to virtue: as it is said (Deuteronomy I:15): 'And I took out of your tribes men, wise and honourable, and appointed them rulers'. And this was aristocracy. But it was democratic in the sense that they were elected from the whole people, for it is said in Exodus (XVIII:21): 'Seek out from the whole people wise men,' etc.; and also in the sense that the people elected them, for it is said in Deuteronomy (I:13): 'Let me have from among you wise and understanding men,' etc. So it is clear that there

was an excellent ordering of authority in the Law [of the Old Testament]. (Aquinas 1948, pp 148-151)

This is what we are accustomed to think of as a Schumpeterian conception of democracy, which should perhaps be relabelled Thomist. Democracy entails the right to elect the sovereign. But while in place, the sovereign's authority is absolute. Clearly, such a doctrine justified the papal constitution, and that of any order that gave its magister sovereign authority. Typically, however, the Dominicans were ahead of the pack. Their constitution foresaw the problem of an incompetent magister. The Master General of the Order held office for life, but the general chapter had the right to impeach him. There were four acceptable reasons:

- 1 Crime
- 2 Causing disunity or harm to the Order
- 3 Inept administration
4. Inability to effectively perform his duties (illness, senility, etc).

Before impeachment, the general chapter was supposed to take one final step, which was to ask for his resignation. If he refused to resign, the impeachment process to formally depose him would take place.

Other self-governing bodies in medieval Europe included communes (such as Venice) and universities (such as Bologna, Paris, Oxford, and Cambridge - including their constituent colleges). They too needed election rules such that they could elect their own heads without outside interference. Their constitutions await detailed study. Just like the papal, imperial, Gilbertine, or Dominican electors, the writers of college constitutions needed a device to prevent a succession crisis. The senior fellow is always an identifiable person (*quae in ecclesia vacante prius fuit recepta*, in Llull's words). The old formula *maior et*

sanior pars could very easily be amended, or corrupted, to *maior et sanior pars*. Seniority can be held to breed wisdom. This confusion is as old as Llull and as new as the Statutes of Nuffield College, Oxford (1958), which entrust the procedure to elect a new Warden to the senior - not to the wisest - of the fellows.

Conclusion

The two modern conceptions of voting both occur in medieval choice theory, and medieval practice. Conception #1 is both epistemic and stochastic. Conception #2 is neither. In first conception, the task of voting is to aggregate imperfect human judgments to the most reliable achievable group judgment of an unknowable true state of affairs. For medieval choice theorists, that unknown state of affairs was the will of God. For Condorcet, reviving choice theory in the 18th century, it was the truth of an allegation that a jury was called upon to decide – hence Condorcet’s formulation has come to be known as his ‘jury theorem’. This conception of voting is still unfamiliar to most voters, but an example where it applies is the task of determining, given that one or more sensors have indicated a fault in a piece of equipment, whether it is likelier that the equipment is faulty or that the sensor(s) have shown a false negative.

But reality kept on intruding. Electors to the papacy abandoned the unanimity rule in favour of the 2/3 rule, but then found that to avoid deadlock they had to impose privation, open voting, and rules against trade-offs and side-payments. The formula ‘*maior et sanior pars*’ was unstable. As early as St Benedict, rule-makers discovered that every defeated minority claimed to be ‘*sanior*’ than the majority. Intriguers like Llull’s fictional archdeacon obviously abounded. Nobody seriously pretended, in Cusanus’ day, that

election of a Holy Roman Emperor was a process of finding the will of God. Rather, it was a process of finding a candidate who was acceptable to a sufficient number of electors. The Borda (Cusanus) rule chooses the candidate who on average ranks highest on electors' schedules. Unfortunately, it is highly manipulable, which is probably why Cusanus recommends voting by secret ballot.

Black's (1958) rediscovery of Condorcet (1785) reintroduced the idea that there are two rival conceptions of voting – as the aggregation of judgments and as the aggregation of interests. In an ordinary election, as Black (1958, p. 163) says, 'the phrase "the probability of the correctness of a voter's opinion"' seems to be without definite meaning. In a jury vote it does have meaning.

The two leading solution concepts for elections and juries faced with $n > 2$ options are the Condorcet rule (with its extension, the Copeland rule), and the Borda rule. The Condorcet rule is 'Choose the option that wins each of its pairwise contests against each of the others'. The Copeland extension is 'Choose the option that scores the largest number of pairwise victories'. The Borda rule is 'Choose the option with the highest average rank'. These solution concepts may be applied to either conception of the nature of elections. For instance, Condorcet found to his embarrassment that where individual voter reliabilities are low, the Borda rule is more likely than the Condorcet rule to find the most probable solution to a jury problem (Black 1958, p. 170). We have shown that they are centuries older than the thinkers whose names they bear. Llull proposed the Copeland rule and may have proposed the Borda rule. Cusanus proposed the Borda rule. It may be too late to rename them, but it is not too late to hail these medieval discoveries.

References

- Aquinas, Thomas. 1948. *Selected Political Writings*. Ed. A.P. D'Entreves, Translated by J.G. Dawson Oxford, Blackwell.
- Barber, B. 1985. *Strong Democracy*. New York: Basic Books.
- Black, D. 1958. *The Theory of Committees and Elections*. Cambridge: Cambridge University Press.
- Bonner, A., ed. 1985. *Selected works of Ramon Llull*. 2 vols. Princeton, NJ: Princeton University Press.
- Caplin, Andrew and Barry Nalebuff. 1988. 'On 64%-Majority Rule' *Econometrica*, 56, 4: 787-814.
- Colomer, J. and McLean, I. 1998. 'Electing popes: approval balloting and qualified-majority rule', *Journal of Interdisciplinary History* 29 (1), 1-22.
- Condorcet, M. J. A. N. C., marquis de. 1785. *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la probabilité des voix*. Paris: Imprimerie Royale.
- Doeberl, M. 1889-94 ed.. *Monumenta Germaniae selecta ab anno 768 usque ad annum 1250*, 5 vols München
- Farquharson, R. 1969. *Theory of Voting*. Oxford: Blackwell.
- Galbraith, G. R. 1925. *The constitution of the Dominican order 1216---1360*. Manchester: Manchester University Press.
- Gardner, M. 1982. *Logic machines and diagrams*. 2nd ed. Brighton: Harvester.
- Gragg, Florence A. and Leona C. Gabel eds. 1959. *Memoirs of a Renaissance Pope. The Commentaries of Pius II*. New York: Putnam's Sons.
- Hägele, G. and Pukelsheim, F. 2001. 'Llull's writings on electoral systems', *Studia Lulliana* 41 (2001), 3-38.

Honecker, M. 1937a. 'Llullus-Handschriften aus dem Besitz des Kardinals Nikolaus von Cues: nebst einer Beschreibung der Llullus-Texte in Trier und einem Anhang über den wiederaufgefundenen Traktat "De Arte Electionis"'. In *Gesammelte Aufsätze zur Kulturgeschichte Spaniens*, edited by H Finke, P. I, § 6, pp. 252-309. Münster in Westfalen: Verlag der Aschendorffschen Verlagsbuchhandlung.

□-----, 1937b. Ramon Lulls Wahlvorschlag Grundlage des Kaiserwahlplanes bei Nikolaus von Cues? *Historisches Jahrbuch* 57:563---74.

Kallen, G., ed. 1964. *Nicolai de Cusa opera omnia*, vol. 14. Hamburg: Felix Meiner. □

Klamler, C. 2005. 'The Copeland rule and Condorcet's principle', *Economic Theory* 25 (3), pp. 745-9.

Lines, M. 1986. 'Approval voting and strategy analysis: A Venetian example'. *Theory and Decision* 20:155--72.

McLean, Iain, and Arnold B. Urken, eds. 1995. *Classics of Social Choice*. Ann Arbor: The University of Michigan Press.

Nuffield College, Oxford. 1958. *Statutes*: Oxford, Oxford University Press for the College.

Peers, E. A., ed. 1926. *Blanquerna: A 13th century romance translated from the Catalan of Raymond Lull*. London: Jarrolds.

Perez Martinez, L. 1959. 'El *Ars notandi* y el *Ars electionis*, dos obras desconocidas de Ramon Llull.' *Estudios Lulianos* 3: 275-278.

Riker, W. H. 1982. *Liberalism against Populism* San Francisco: W. H. Freeman.

Riker, W. H. 1986. *The Art of Political Manipulation*. New Haven, CT: Yale University Press.

Robinson, I. S. 1990. *The Papacy 1073-1198. Continuity and Innovation*. Cambridge: Cambridge University Press.

- Ruffini Avondo, Edoardo. 1925. 'Il principio maggioritario nella storia del Diritto Canonico', *Archivio □ Giuridico 'Filippo Serafini'*, 1925, 4, IX, 15-G7. □
- Saari, Donald G. 1994. *Geometrv of Voting*. Berlin: Springer. □
- Sigmund, P. E. 1963. *Nicolaus of Cusa and medieval political thought*. Cambridge, MA: Harvard University Press.
- . ed. 1991. *Nicolaus of Cusa: **The Catholic Concordance***. Cambridge: Cambridge University Press.
- Ullmann, Walter. 1975. *Law and Politics in the Middle Ages. An Introduction to the Sources of Medieval Political Ideas*. Cambridge: Cambridge University Press.
- Vauchez, André (dir.). 1990. *Apogée de la Papauté et expansion de la Chrétienté (1054-1274)*. Paris: Desclee, 'Histoire du Christianisme' (dir. JM. Mayeur et al.), vol.5.

¹ This could mean 'guardian' in any of several senses, e.g., one appointed to watch over good order generally, or more specifically to supervise votes and elections.